

Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes

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Abstract. In this paper I define a new type of pairs of primes, id est the Smarandache-Germain pairs of primes, notion related to Sophie Germain primes and also to Smarandache function, and I conjecture that for all pairs of Sophie Germain primes but a definable set of them there exist corespondent pairs of Smarandache-Germain primes. I also make a conjecture that attributes to the set of Sophie Germain primes but a definable subset of them a corespondent set of smaller primes, id est Coman-Germain primes.

Conjecture 1:

For any pair of Sophie Germain primes $[p_1, p_2]$ with the property that $S(p_1 - 1)$ is prime, where S is the Smarandache function, we have a corresponding pair of primes $[S(p_1 - 1), S(p_2 - 1)]$, which we named it Smarandache-Germain pair of primes, with the property that between the primes $q_1 = S(p_1 - 1)$ and $q_2 = S(p_2 - 1)$ there exist the following relation: $q_2 = n \cdot q_1 + 1$, where n is non-null positive integer.

Note:

For a list of Sophie Germain primes see the sequence A005384 in OEIS. For the values of Smarandache function see the sequence A002034 in OEIS.

Verifying the Conjecture 1:

(for the first 26 pairs of Sophie Germain primes)

- : For $[2, 5]$ we have $S(2 - 1) = 1$, not prime;
- : For $[3, 7]$ we have $S(3 - 1) = 2$, not odd prime;
- : For $[5, 11]$ we have $S(5 - 1) = 4$, not prime;
- : For $[11, 23]$ we have $[S(10), S(22)] = [5, 11]$
and $5 \cdot 2 + 1 = 11$;
- : For $[23, 47]$ we have $[S(22), S(46)] = [11, 23]$
and $11 \cdot 2 + 1 = 23$;
- : For $[29, 59]$ we have $[S(28), S(58)] = [7, 29]$
and $7 \cdot 4 + 1 = 29$;
- : For $[41, 83]$ we have $[S(40), S(82)] = [5, 41]$
and $5 \cdot 8 + 1 = 41$;
- : For $[53, 107]$ we have $[S(52), S(106)] = [13, 53]$
and $13 \cdot 4 + 1 = 53$;
- : For $[83, 167]$ we have $[S(82), S(166)] = [41, 83]$
and $41 \cdot 2 + 1 = 83$;

: For [89, 179] we have $[S(88), S(178)] = [11, 89]$
 and $11 \cdot 8 + 1 = 89$;
 : For [113, 227] we have $[S(112), S(226)] = [7, 113]$
 and $7 \cdot 16 + 1 = 113$;
 : For [131, 263] we have $[S(130), S(262)] = [13, 131]$
 and $13 \cdot 10 + 1 = 131$;
 : For [173, 347] we have $[S(172), S(346)] = [43, 173]$
 and $43 \cdot 4 + 1 = 173$;
 : For [179, 359] we have $[S(178), S(358)] = [89, 179]$
 and $89 \cdot 2 + 1 = 179$;
 : For [191, 383] we have $[S(190), S(382)] = [19, 191]$
 and $19 \cdot 10 + 1 = 191$;
 : For [233, 467] we have $[S(232), S(466)] = [29, 233]$
 and $29 \cdot 8 + 1 = 233$;
 : For [239, 479] we have $[S(238), S(478)] = [17, 239]$
 and $17 \cdot 14 + 1 = 239$;
 : For [251, 503] we have $S(250 - 1) = 15$, not prime;
 : For [281, 563] we have $[S(280), S(562)] = [7, 281]$
 and $7 \cdot 40 + 1 = 281$;
 : For [293, 587] we have $[S(292), S(586)] = [73, 293]$
 and $73 \cdot 4 + 1 = 293$;
 : For [359, 719] we have $[S(358), S(718)] = [179, 359]$
 and $179 \cdot 2 + 1 = 359$;
 : For [419, 839] we have $[S(418), S(838)] = [19, 419]$
 and $19 \cdot 22 + 1 = 419$;
 : For [431, 863] we have $[S(430), S(862)] = [43, 431]$
 and $43 \cdot 10 + 1 = 431$;
 : For [443, 887] we have $[S(442), S(886)] = [17, 443]$
 and $17 \cdot 26 + 1 = 443$;
 : For [491, 983] we have $S(491 - 1) = 14$, not prime;
 : For [509, 1019] we have $[S(508), S(1018)] = [127, 509]$
 and $127 \cdot 4 + 1 = 509$.

Conjecture 2:

There exist an infinity of Smarandache-Germain pairs of primes.

Note:

It can be seen that $q_2 = S(p_2 - 1) = p_1$ and also n is often a power of the number 2, so I make a new conjecture:

Conjecture 3:

For any p Sophie Germain prime with the property that $S(p - 1)$ is prime, where S is the Smarandache function, one of the following two statements is true:

1. there exist m non-null positive integer such that $(p - 1)/(2^m) = q$, where q is prime, $q \geq 5$;
2. there exist n prime and m non-null positive integer such that $(p - 1)/(n \cdot 2^m) = q$, where q is prime, $q \geq 5$.

Note: we call the primes q from the first statement Coman-Germain primes of the first degree; we call the primes q from

the second statement Coman-Germain primes of the second degree.

Verifying the Conjecture 3:

(for the first 21 Sophie Germain primes with the property showed)

The first statement:

- : For $p = 11, 23, 83, 179$ we have $m = 1$
and $q = 5, 11, 41, 89$;
- : For $p = 29, 53, 173, 293, 509$ we have $m = 2$
and $q = 7, 13, 43, 73, 127$;
- : For $p = 41, 89, 233$ we have $m = 3$
and $q = 5, 11, 29$;
- : For $p = 113$ we have $m = 4$
and $q = 7$.

The second statement:

- : For $p = 131, 191, 431$ we have $(m, n) = (1, 5)$
and $q = 13, 19, 43$;
- : For $p = 239$ we have $(m, n) = (1, 7)$
and $q = 17$;
- : For $p = 281$ we have $(m, n) = (3, 5)$
and $q = 7$;
- : For $p = 419$ we have $(m, n) = (1, 11)$
and $q = 19$;
- : For $p = 443$ we have $(m, n) = (1, 13)$
and $q = 17$.

Conjecture 4:

There exist an infinity of Coman-Germain primes of the first degree.

Conjecture 5:

There exist an infinity of Coman-Germain primes of the second degree.

Notes:

We have the following sequence of Smarandache-Germain pairs of primes:

[5, 11], [11, 23], [7, 29], [5, 41], [13, 53], [41, 83], [11, 89], [7, 113], [13, 131], [43, 173], [89, 179], [19, 191], [29, 233], [17, 239], [7, 281], [73, 293], [179, 359], [19, 419], [43, 431], [17, 443], [127, 509] (...).

We have the following sequence of Coman-Germain primes of the first degree:

5, 11, 7, 5, 13, 41, 11, 7, 13, 43, 89, 29, 73, 179, 127 (...).

We have the following sequence of Coman-Germain primes of the second degree:

13, 19, 17, 7, 19, 43, 17 (...).